*Int. J. Heat Mass Transfer*. Vol. 20, pp. 1351–1354. Pergamon Press 1977. Printed in Great Britain

# TEMPERATURE DISTRIBUTIONS IN A SYSTEM OF TWO CYLINDRICAL BODIES

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(*Received* 3 *February* 1976)

**Abstract**—Consideration is made of transient heat transfer in a cylindrical fuel element of the nuclear reactor. The fuel rod, whose power dissipation is assumed to take place in the fuel element by a sinusoidal pulse, and cladding are separated by a thin layer. The conjugate boundary-value problem involving the equation for heat conduction in the fuel rod and cladding, and boundary conditions with regard for non-ideal thermal contact is solved by the Laplace transformation. The results on a temperature distribution for uranium dioxide fuel rod and zirconium cladding when cooled by liquid sodium are presented graphically.

## NOMENCLATURE

$a_1$ $a_2$, thermal diffusivities of fuel rod and cladding material;
$C^{111}$, heat capacity/volume;
$Bi$, $= h\delta/\lambda_2$, Biot number;
$Fo$, $= a_1 t/R^2$, Fourier number;
$h$, heat-transfer coefficient;
$I_k(x)$, modified Bessel's function of the first kind of order $k$ for argument $x$;
$J_k(x)$, Bessel's function of first kind of order $k$ for argument $x$;
$K_q$, $= K_a K_\delta$ dimensionless parameters;
$K_a$, $= \sqrt{a_1/a_2}$;
$K_\delta$, $= \delta/R$, relative thickness of the cladding;
$K_\lambda$, $= \lambda_1/\lambda_2$, relative thermal conductivity;
$K_h$, $= K_\lambda/K_a$, relative heat penetration coefficient;
$l$, phase period;
$m$, thermal resistance;
$r$, radial variable;
$R$, radius of the fuel element;
$s$, Laplace parameter;
$T_1$, $T_2$, temperature distributions in the fuel rod and cladding;
$T_{10}$, $T_{20}$, initial temperatures of the fuel rod and cladding;
$T_c$, coolant temperature;
$\Delta T_{ref}$, $= \left.\frac{\phi''' R^2}{c'''}\right|_{Fo=0}$, unit step function;
$U(Fo-\lambda/l)$, amplitude of heat source strength;
$x$, spatial co-odrinate in cladding;
$\chi$, $= \frac{\chi}{R}$, $\xi = \frac{\chi}{\delta}$, dimensionless space variable;
$\theta_1$, $= \frac{T_1 - T_c}{\Delta T_{ref}}$, $\theta_2$, $= \frac{T_2 - T_c}{\Delta T_{ref}}$, dimensionless temperature distributions;
$\theta_{10}, \theta_{20}$, initial temperature distributions in dimensionless form;
$\phi'''$, heat source strength;
$\lambda_1, \lambda_2$, thermal conductivities;
$\delta$, cladding thickness.

## INTRODUCTION

THE MOST general system describing the transient heat transfer between a fuel element of a nuclear reactor and coolant flowing past it is described by a complete set of energy balance equations for the fuel element, cladding and the coolant together with the hydrodynamical equation for the system. Such a conjugate system leads to various complications for analysis purposes and from a realistic point of view it is meaningful to discard the hydrodynamical equation [1, 2]. Further as a result of good heat-transfer properties of liquid metals, the surface of the fuel rods of sodium cooled reactors is only slightly hotter than the coolant. Therefore, the accurate knowledge of the heat transfer between the fuel rods and the coolant is of little importance in the thermal design of such fuel rods and a designer is mainly concerned with the transient histories of the temperature distribution of the fuel rods and the cladding material of a fuel reactor.

## MODEL DESCRIPTION

The rod is considered to be made of fuel, bond and cladding having homogeneous materials with constant properties. The bond and cladding are generally very thin layers which are considered flat under the first degree of approximation. Power dissipation is assumed to take place in the fuel element by a sinusoidal pulse. This assumption is consistent with the physics of the problem because in a nuclear reactor, the actual neutron population is oscillating about the average value. These oscillations will affect the reactivity, and it

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is convenient to regard the power dissipation as being induced by the oscillating reactivity. With these assumptions, the temperature field in the fuel element satisfies the Fourier heat-conduction equation:

$$\frac{\partial\theta_1}{\partial Fo} = \frac{1}{\chi}\frac{\partial}{\partial\chi}\left(\chi\frac{\partial\theta_1}{\partial\chi}\right) + [1-U(F-\pi/l)]\sin lFo \quad (1)$$

and in the cladding,

$$\frac{\partial\theta_2}{\partial Fo} = \frac{1}{K_q^2}\frac{\partial^2\theta_2}{\partial\xi^2} \quad (2)$$

where the dimensionless variables and criteria are listed in the nomenclature. In writing equations (1) and (2), the following assumptions are also applied: (a) no axial conduction in any of the materials; (b) azimuthal symmetry, and (c) no circumferential heat transfer in the gap.

Associated with equations (1) and (2) the initial and boundary conditions are

$$\theta_1(\chi,0) = \theta_{10}, \quad \theta_2(\xi,0) = \theta_{20} \quad (3)$$

$$\frac{\partial\theta_2(1,Fo)}{\partial\xi} + Bi\theta_2(1,Fo) = 0. \quad (4)$$

The fuel rod and cladding are separated by a thin layer of the bonding material with low thermal conductivity which offers a thermal resistance in heat propagation from fuel rod to cladding material and establishes non-ideal thermal contact. For the non-ideal thermal contact between the fuel element and the cladding, the continuity conditions can be put in slightly modified form [3].

$$K_j K_\alpha \frac{\partial\theta_1(1,Fo)}{\partial\chi} = \frac{\partial\theta_2(0,Fo)}{\partial\xi} \quad (5)$$

and

$$\theta_1(1,Fo) = \theta_2(0,Fo) - M\frac{\partial\theta_1(1,Fo)}{\partial\chi} \quad (6)$$

where $M = m\lambda_1/R_1$, $m$ being the non-ideal thermal contact coefficient.

The set of equations (1)–(6) forms a conjugate boundary value problem which can be tackled by the method of Laplace transformation for analytical solutions which is well treated in the treatise of Luikov [4] and Carslaw and Jaeger [5]. Tong and Weisman [6] have also obtained analytical solution of the problem relating to a step change by finite Fourier transform. Following the treatments of the present authors [7, 8] the analytical solutions can be written under Laplace transformation as

$$\bar{\theta}_1 - \frac{\theta_{10}}{s} = -\frac{l[1+\exp(s\pi/l)]}{s(l^2+s^2)}\left|1-\frac{N_A}{V(s)}I_0(q_1\chi)\right| + \frac{1}{sV(s)}[(\theta_{20}-\theta_{10})N_A - \theta_{20}Bi]I_0(q_1\chi) \quad (7)$$

and

$$\bar{\theta}_2 - \frac{\theta_{20}}{s} = -\frac{K_b lI_1(q_1)}{s(l^2+s^2)V(s)}[1+\exp(-s\pi l)N_B - \frac{(\theta_{20}-\theta_{10})K_b N_B I_1(q_1)}{sV(s)} - \frac{Bi\theta_{20}}{sV(s)}[(I_0(q_1)+Mq_1I_1(q_1)) \times \cosh q_2\xi + K_b I_1(q_1)\sinh q_2\xi] \quad (8)$$

where

$$N_A = q_2\sinh q_2 + Bi\cosh q_2 \quad (9)$$

$$N_B = q_2\cosh q_2(1-\xi) + Bi\sinh q_2(1-\xi) \quad (10)$$

$$V(s) = [I_0(q_1)+Mq_1I_1(q_1)] \times [q_2\sinh q_2 + Bi\cosh q_2] + K_b I_1(q_1)[q_2\cosh q_2 + Bi\sinh q_2] \quad (11)$$

and

$$\sqrt{s} = q_1, \quad K_q\sqrt{s} = q_2.$$

In transition from the variable $s$ to $Fo$, the residue theorem of the complex analysis can be applied. To determine the zeros of the function $V(s)$, we set it to zero, this gives an infinite number of simple roots $s = s_n$. To make the values of $s_n$ more precise, we replace $s = -\mu^2$, i.e. $q_1 = i\mu$, $q_2 = iv$, $v = K_q\mu$ in the expression of $V(s)$ giving rise,

$$(-v\sin v + Bi\cos v)[J_0(\mu) - M\mu J_1(\mu)] - K_b J_1(\mu)[v\cos v + Bi\sin v] = 0 \quad (12)$$

or,

$$v - n\pi = \arctan\frac{Bi}{v} - \arctan\frac{K_b}{J_0(\mu)/J_1(\mu) - M\mu}. \quad (13)$$

Inverting the expressions (7) and (8), we obtain

$$\theta_1 = -\frac{1}{l}(1-\cos lFo) + 2\sum_{n=1}^{\infty}(1/\mu_n\psi_n) \times [Bi\theta_{20} - (\theta_{20}-\theta_{10})N_{An}] \times J_0(\mu_n\chi)\exp(-\mu_n^2 Fo) + \int_0^{Fo}\sin l(Fo-u) \times [1-2(1/\mu_n\psi_n)N_{An}J_0(\mu_n\chi)\exp(-\mu_n^2 u)]\,du \quad (14)$$

and

$$\theta_2 = -2(K_b l)\sum_{n=1}^{\infty}(1/\mu_n\psi_n)N_{Bn} \times \sin l(Fo-u)J_1(\mu_n)\exp(-\mu_n^2 u)\,du + 2\sum_{n=1}^{\infty}(1/\mu_n\psi_n)[K_b(\theta_{10}-\theta_{20})N_{Bn}J_1(\mu_n) + Bi\theta_{20}\{(J_0(\mu_n) - M\mu_n J_1(\mu_n)\cos v_n\xi - K_b J_1(\mu_n)\sin v_n\xi\}]\exp(-\mu_n^2 Fo) \quad (15)$$

where

$$N_{An} = -v_n \sin v_n + Bi \cos v_n \quad (16)$$

$$N_{Bn} = v_n \cos[v_n(1-\xi)] + Bi \sin[v_n(1-\xi)] \quad (17)$$

and

$$\psi_n = -(Bi \cos v_n - v_n \sin v_n)\{J_1(\mu_n)(1+K_\lambda K_\delta) + M\mu_n J_0(\mu_n) + K_\lambda K_\delta\, '_1(\mu_n) \cos v_n + (Bi \sin v_n + v_n \cos v_n)[(K_b + K_q)J_0(\mu_n) - (MK_q\mu_n + K_b/\mu_n)J_1(\mu_n)] + K_b \sin v_n[J_0(\mu_n) - M\mu_n J_1(\mu_n)\}. \quad (18)$$

## DISCUSSION OF RESULTS

Consider a sodium-cooled, cylindrical fuel rod of uranium-dioxide with zirconium cladding. Its dimensions and properties [9, 10] are:

Table 1. Material properties

| | Radius (cm) | Thermal conductivity (cal/s cm °C) | Thermal diffusivity ($cm^2/s$) |
|---|---|---|---|
| Uranium dioxide | $R = 0.28$ | 0.0955 | 0.0147 |
| Zirconium alloy | $\delta = 0.05$ | 0.4795 | 1.9719 |

This determines the values of the various parameters as $K_\delta = 0.175$, $K_a = 0.863$, $K_\lambda = 0.1991$, $K_b = 0.2307$. Further initially let $\theta_1 = 12.0$ and $\theta_2 = 0.4$.

The transcendental equation (12) or (13) is solve numerically for the characteristic root for different values of $M$ and $Bi$.

The first two roots are presented in Table 2.

The expression for temperature distribution, $\theta_i(i = 1, 2)$ contains a highly convergent series in the generalised time variable $Fo$, so after a certain value of $Fo$ (say $Fo > Fo_1$) only first two terms of the series are sufficient to give the accurate result. This is clear since $\mu_1 < \mu_2 < \mu_3 \ldots \mu_n$ and consequently the exponential function $\exp(-\mu_n^2 Fo)$ quickly decreases with an increase in $Fo$ for example, at $Fo_1 = 0.2$, $\exp(-\mu_1^2 Fo) = 0.3354$ and $\exp(-\mu_2^2 Fo) = 0.0033$ for a set of values $M = 0$, $Bi = 5$, which shows the consideration of second term only affects in third decimal place of calculation.

Table 2. Characteristic roots

| $M$ | $Bi$ | $\mu_1$ | $\mu_2$ |
|---|---|---|---|
| 0 | 5 | 2.337003 | 5.344689 |
| | 10 | 2.328188 | 5.315596 |
| | 15 | 2.325212 | 5.305088 |
| | 20 | 2.323718 | 5.299672 |
| | 25 | 2.322818 | 5.296369 |
| | 30 | 2.322217 | 5.294145 |
| | 35 | 2.321788 | 5.292546 |
| 1 | 5 | 1.404775 | 4.605086 |
| | 10 | 1.401102 | 4.594266 |
| | 15 | 1.399878 | 4.590472 |
| | 20 | 1.399266 | 4.588539 |
| | 25 | 1.398889 | 4.587366 |
| | 30 | 1.398654 | 4.586579 |
| | 35 | 1.398479 | 4.586014 |
| 2 | 5 | 0.887201 | 4.293626 |
| | 10 | 0.886104 | 4.289646 |
| | 15 | 0.885738 | 4.288253 |
| | 20 | 0.885555 | 4.287545 |
| | 25 | 0.885446 | 4.287114 |
| | 30 | 0.885373 | 4.286826 |
| | 35 | 0.885321 | 4.286619 |

The temperature distributions for the fuel rod and cladding material are shown graphically. The influences of surface resistance on cladding material for different values of "$Bi$" are shown in Fig. 1. It is observed that the surface resistance does not have considerable effect on the cladding temperature. Further an increase in the Biot modulus "$Bi$" causes the cladding surface temperature to moe closely approach

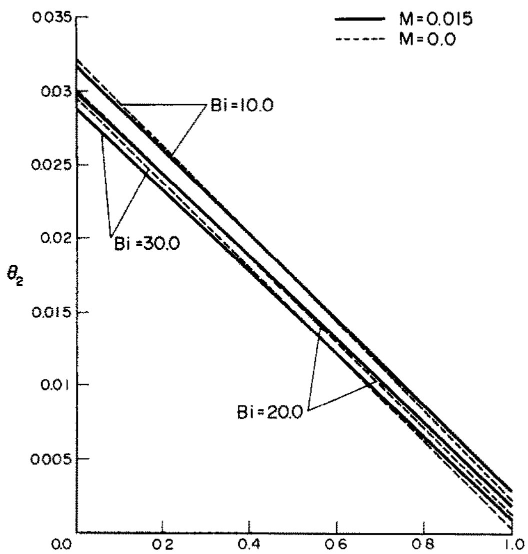


FIG. 1. Influence of surface resistance on cladding ($Fo = 0.6$).

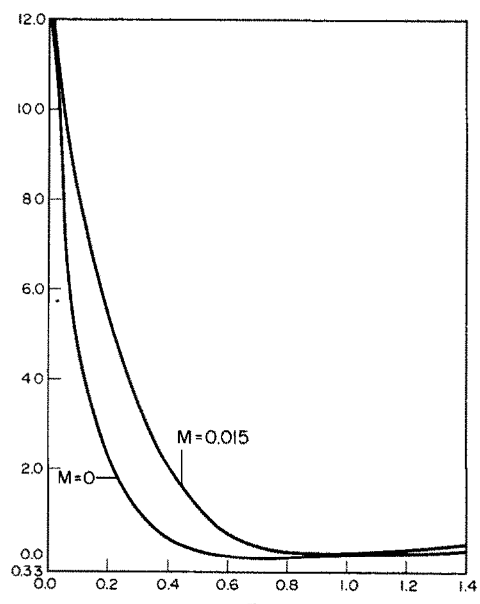


FIG. 2. Influence of surface resistance on fuel rod ($Bi = 10.0$), $x = 0$.

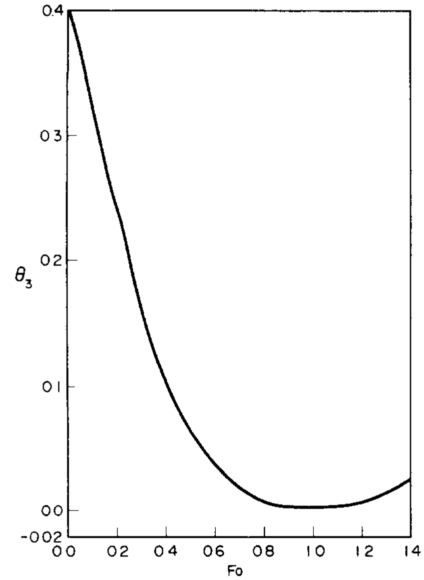


FIG. 3. Temperature histories of cladding surface ($Bi$ = 10.0), $M = 0$.

the coolant temperature. Figures 2 and 3 describe the temperature distributions of fuel rod (at $\chi = 0$) and cladding material (at $\xi = 0$) in transient states.

From Fig. 2, it is observed that the presence of surface resistance on fuel rod has a considerable role in transient state. This reduces the heat removal rate considerably in the interval ($0 \leqslant Fo \leqslant 0.8$) and afterwards a pseudosteady state is arrived in which the non-ideal thermal contact parameter becomes of little importance.

*Acknowledgements*—One of the authors (K. N. Shukla) is grateful to Mr. S. F. Hussain for rendering his services in the computational work of this paper. He is also grateful to Dr. C. A. Reddy for his encouragement throughout the work.

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## DISTRIBUTION DE TEMPERATURE DANS UN SYSTEME DE DEUX CORPS CYLINDRIQUES

**Résumé**—On considère le transfert thermique variable dans un élément combustible cylindrique de réacteur nucléaire. La barre combustible, à l'intérieur de laquelle la puissance dissipée a lieu par pulsation sinusoïdale, et la gaine sont séparée par une couche mince. Le problème couplé, décrit par l'équation de la conduction thermique dans la barre combustible et la gaine avec des conditions aux limites tenant compte d'un contact thermique imparfait, est résolu par la transformation de Laplace. Les résultats relatifs à la distribution de température dans une barre de bioxyde d'uranium, une gaine en zirconium et un refroidissement par sodium liquide, sont présentés graphiquement.

## DIE TEMPERATURVERTEILUNG IN EINEM SYSTEM AUS ZWEI ZYLINDRISCHEN KÖRPERN

**Zusammenfassung**—Es wird der instationäre Wärmeübergang in einem zylindrischen Brennelement eines Kernreaktors untersucht. Der Brennstab, dessen Energiedissipation im Brennelement als sinusförmig angenommen wird, ist durch eine dünne Schicht von der Hülse getrennt. Das konjugierte Randwertproblem, das die Wärmeleitgleichung für Brennstab und Hülse sowie die Randbedingungen für nicht-idealen thermischen Kontakt beinhaltet, wird mit Hilfe der Laplace-Transformation gelöst. Die Ergebnisse für die Temperaturverteilung in einem mit flüssigem Natrium gekühlten Urandioxid-Brennstab mit Zirkonhülse werden grafisch wiedergegeben.

## ТЕМПЕРАТУРНЫЕ РАСПРЕДЕЛЕНИЯ В СИСТЕМЕ ДВУХ ЦИЛИНДРИЧЕСКИХ ТЕЛ

**Аннотация** — Рассматривается нестационарная задача теплопроводности в цилиндрическом ТВЭЛе ядерного реактора. Между рабочим телом, диссипация энергии в котором полагается изменяющейся со временем синусоидально, и кожухом помещается тонкая прокладка. Сопряженная краевая задача, включающая уравнения теплопроводности в рабочем теле и кожухе, а также граничные условия с учетом неидеального теплового контакта решается с помощью преобразования Лапласа. Графически представлены результаты расчетов распределения температур для рабочего тела из двуокиси урана и циркониевого кожуха при охлаждении жидким натрием.